TOPIC: UNIFIED CODE FOR COLLOCATION MULTISTEPS METHOD FOR SOLVING STIFF SYSTEM OF BOUNDARY VALUE PROBLEM

Key points:

1. Multistep:

One of the major disadvantage of R-K methods is that as the order of the method is increased the required number of function evaluation also increases hence the need for multistep method.

Multistep methods are a class of numerical techniques used for solving ordinary differential equations (ODEs), including both Initial Value Problems (IVPs) and Boundary Value Problems (BVPs).

For a single step method like R-K method, Euler method, which uses only the initial value, usually .

Multistep method uses information from several previous steps to compute the solution at the current step

Key Characteristics:

* Using information from previous steps
* Higher order of accuracy
* Increased efficiency
* Stability: Some multisteps method can be unstable particularly for stiff differential equation. These methods can struggle to keep track of these rapid change in situation and lead to inaccurate or even divergent solutions.

Multisteps method can be explicit or implicit i.e Explicit methods directly compute the solution at the next step, while implicit methods involve solving an equation to determine the next step.,Examples of multisteps method includes;

1. Adams-Bashforth method (Explicit method)
2. Adams-Multon method (Implicit method)
3. Eulers method (2nd Step)
4. Boundary Value Problems:

**A special type of ODE with additional conditions imposed at specific points, called boundaries.** These conditions dictate the behavior of the solution at the edges of the problem's domain. These boundaries dictate the behavior of the solution, acting like fences that guide the flow of information within the problem. Think of a swimming pool: the pool walls act as boundaries, confining the water and defining its shape. Similarly, BVPs have boundaries that dictate how the solution, like the water level in the pool, behaves within the problem's domain.

BVPs play a crucial role in mathematical modeling by providing a framework to describe phenomena that depend not only on the initial state but also on conditions at different points across the domain.

They have their applications in many fields of sciences such as Heat transfer, Chemical reactions, fluid dynamics, Material sciences, medicinal sciences and many more fields.

Boundary value problems exhibits a special characteristic called “Stiffness”, stiff BVPs have solutions that can change rapidly and dramatically, making them challenging for traditional numerical methods.

Methods used for solving BVPs are: Shooting method, Finite element method, Finite difference method, Collocation method among all other method. The shooting method works by first reducing the BVP to an Initial Value Problem (IVP), then one/two initial value guesses are made. The IVPs are then solved using an iterative solution, and this process is then repeated until the second boundary condition is reached to a satisfactory level.

We will focus more on collocation method for solving boundary value problems,

The collocation method is a numerical technique for solving boundary value problems (BVPs) by approximating the solution as a linear combination of basis functions and enforcing the satisfaction of the differential equations at specific collocation points within the domain. Here's a step-by-step explanation of the collocation method for solving BVPs:

1. Formulating the BVP:

Subject to the boundary conditions

The goal is to find the solution that satisfies both the differential equation and the given boundary conditions.

1. Approximate solution:

Represent the approximate solution in the form:

1. Enforce equations at collocation points

Choose a set of collocation points xi within the domain (*a*,*b*). Enforce the satisfaction of the differential equation at these collocation points:

This leads to a system of algebraic equations

1. **System of Equations:**
   * The collocation conditions lead to a system of nonlinear algebraic equations:

*G*(*c*1​,*c*2​,...,*cN*​)=0

* + *G* is a vector function representing the residual.

1. **Solve for Coefficients:**
   * Utilize a numerical solver, such as Newton's method, to solve the system of equations for the coefficients *ci*​.
2. **Reconstruct the Solution:**
   * Once the coefficients are determined, reconstruct the approximate solution *y*(*x*) using the basis functions and coefficients:

3. Stiff System

J. D. Lambert defines stiffness as follows:

If a numerical method with a finite region of absolute stability, applied to a system with any initial conditions, is forced to use in a certain interval of integration a step length which is excessively small in relation to the smoothness of the exact solution in that interval, then the system is said to be *stiff* in that interval.

They are differential equation for which certain numerical methods for solving the equation are numerically unstable, unless the step size is taken to be extremely small.

When integrating a differential equation numerically, one would expect the requisite step size to be relatively small in a region where the solution curve displays much variation and to be relatively large where the solution curve straightens out to approach a line with slope nearly zero. For some problems this is not the case. In order for a numerical method to give a reliable solution to the differential system sometimes the step size is required to be at an unacceptably small level in a region where the solution curve is very smooth. The phenomenon is known as *stiffness*. In some cases there may be two different problems with the same solution, yet one is not stiff and the other is. The phenomenon cannot therefore be a property of the exact solution, since this is the same for both problems, and must be a property of the differential system itself. Such systems are thus known as *stiff systems*.

4. System

A system of ordinary differential equations (ODEs) is a collection of differential equations that involve multiple dependent variables and their derivatives with respect to an independent variable. These systems often arise in various scientific and engineering applications to model complex dynamic processes involving interactions between different components.

